NaSTech
Journal of Natural Sciences and Technologies
2025, 4(2), pp. 431-435, DOI: 10.5281/zenodo.18116456

MANAGEMENT MODELS BASED ON FUZZY CLASSIFICATION OF TECHNOLOGICAL OBJECT STATES

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ABSTRACT

The article proposes a principle of situational management implemented on the basis of fuzzy classification of states of technological objects using fuzzy cluster analysis. The results obtained in the work are the theoretical basis for the development of models for technological process management in the production of technical automation equipment in conditions of insufficient and/or fuzzy information.
Keywords: management models, state classification, technological object, multi-valued (fuzzy) logic.

1. INTRODUCTION

Currently, a new direction in management theory is emerging—management algorithms based on fuzzy models of intelligent controllers and control systems. Work is underway on the practical implementation of fuzzy controllers, expert systems, and control systems in industrial and non-industrial spheres. Thanks to the use of fuzzy controllers, it has become possible to move to more universal management methods, which allows expanding the class of tasks solved in process management systems and increasing the economic efficiency of these systems.

However, existing publications on fuzzy management in the technology of technical means of automation (TMA) are mainly of a review and analytical nature and do not contain a sufficient theoretical basis for controlling technological processes (TP) of TMA production in conditions of insufficient and fuzzy information.

At the same time, the current level of accumulated results in such areas as decision-making theory, fuzzy set theory, operations research, artificial intelligence, and simulation modeling allows us to speak of the possibility of a more in-depth implementation of the theoretical results obtained in these areas into the practice of organizing intelligent management of TP production in conditions of uncertainty. However, when creating specific models of TP management, general theoretical provisions require significant refinement and development in this area.

2. MAIN MATERIAL PRESENTATION

For most multidimensional technological objects (TO), the formation of control models cannot be carried out without the use of heuristic procedures. The intensive use of such procedures reflects a characteristic feature of the systematic approach to the study of complex processes and systems [1].

Let us assume that the efficiency of the TO management process is assessed by some integral indicator

$$\boldsymbol{J} = \boldsymbol{F_1}(\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{w}, \boldsymbol{u}), \tag{1}$$

where $\boldsymbol{x} = (\boldsymbol{x_1}, \boldsymbol{x_2}, \dots, \boldsymbol{x_m})$ is the set of TO input parameters, $\boldsymbol{x} \in \boldsymbol{X}$;
$\boldsymbol{y} = (\boldsymbol{y_1}, \boldsymbol{y_2}, \dots, \boldsymbol{y_n})$ is the set of TO output parameters, $\boldsymbol{y} \in \boldsymbol{Y}$;
$\boldsymbol{w} = (\boldsymbol{w_1}, \boldsymbol{w_2}, \dots, \boldsymbol{w_k})$ is the set of uncontrolled external influences, $\boldsymbol{w} \in \boldsymbol{W}$;
$\boldsymbol{u} = (\boldsymbol{u_1}, \boldsymbol{u_2}, \dots, \boldsymbol{u_l})$ is the set of control influences, $\boldsymbol{u} \in \boldsymbol{U}$.

Then the task of identifying the maintenance management law is to determine *u*:

$$\boldsymbol{u} = \boldsymbol{F_2}(\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{w}), \tag{2}$$

which, with the existing values of $\boldsymbol{x} \in \boldsymbol{X}$, $\boldsymbol{y} \in \boldsymbol{Y}$, $\boldsymbol{w} \in \boldsymbol{W}$ leads to the optimal value of *J*.

Within the framework of a systematic approach, one of the most effective methods for solving problems (1) – (2) is the situational management method [2-4].

The basic idea of this method is based on the assumption that the set of management strategies that satisfy problems (1) – (2) is always smaller than the set of situations that arise at the management object. This means that among the set of possible TO states, there must be subsets, each of which requires the same management law.

It follows that if it is possible to classify the set *S* of possible states of the TO, i.e., it is possible to form such subsets $\boldsymbol{s_i} \subset \boldsymbol{S}$ that

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$$\bigcup_{i=1}^{N} s_i = S; \quad s_i \bigcap s_j \neq \emptyset, \quad i \neq j, i, j = 1, \dots, N,$$

each of which corresponds to its own control law, then the task (1) – (2) will be significantly simplified, since determining the optimal control strategy for an arbitrary element $x_k \in s_j$, will, in fact, mean determining this strategy for all $x \in s_j$.

A stricter definition can be formulated as follows.

Let the state of TO be characterized by a set of controlled parameters *x*. Let *X* be the range of possible values of vector *x*, and $P = (p_1, p_2, \dots, p_n)$ let be the finite set of labels indexing the set of possible TO control laws.

It is necessary to define such a mapping (classification model) f : X → P, whose equivalence classes are interpreted as groups of similar TO states.

From a physical point of view, the possibility of forming a mapping *f* implies that the TO state reflects similar patterns of its functioning, which, in turn, require similar control strategies.

There are two approaches to identifying *f*. If, for a certain subset of elements $X' \subset X$, their belonging to a certain class is known in advance, i.e., it is known that

$$f(x) = p_i, \ \forall x \in X', \ i = 1, \dots, N, \tag{3}$$

it is said that determining the value of the mapping *f* on an arbitrary element $x \in X/X'$ is a supervised classification task (pattern recognition task).

If information of type (3) is missing, the calculation of the value of the mapping on an arbitrary element $x \in X$ is referred to as an unsupervised classification task (automatic classification or cluster analysis task).

According to the above definitions, the models for solving these tasks are called classification models (supervised and unsupervised).

Under conditions of uncertainty, the classification model f cannot be accurately restored, especially in the early stages of modeling. Therefore, for real-world tasks, the situational management method of TO includes the following steps:

- states (situations) $x_1, x_2, \dots, x_n$ of TO at moments in time $t_1, t_2, \dots, t_n$, grouped in an optimal way into classes of output situations $s_1, s_2, \dots, s_n$. An approximate representation of the classification model *f* is formed;
- the situation x_{n+1} observed at TO at time t_{n+1}, either belongs to the class s_j $(j = 1, \dots, N)$ of situations closest to it, for which a management strategy is established using the mapping *f*, or "gives rise" to the creation of a new class of situations s_{N+1}, for which the management strategy does not coincide with any of the strategies identified in the previous stage.

Thus, the main content of the situational management method consists in forming "homogeneous" classes of states.

When classifying experimental observations, the following types of models are most often used [5, 6]:

- models of splitting mixtures of probability distributions (classification models with training);
- classical cluster analysis models (classification models without training);
- hierarchical classification models.

When analyzing the possibility of using the listed models to solve classification problems that arise in maintenance management tasks, it is necessary, first of all, to note the following [7].

In mixture splitting models, the form of the corresponding splitting function is considered known (with the accuracy of specific parameter values). However, the assumption of any distribution law is always associated with the question of the adequacy of the predicted law to the actual distribution. In conditions of uncertainty, such a question remains without a satisfactory answer. With this approach, we have to deal with multimodal distribution densities, the accurate restoration of which is practically impossible. In addition, the type of probability distribution function can only be specified for quantitatively measurable characteristics, which also does not correspond to the specifics of TO as a modeling object.

As for hierarchical classification and cluster analysis models, the most significant drawback of such models (as well as most traditional automatic classification models) is related to the underlying binary logic of membership and the principle of unambiguous (clear) division of TO states into classes. Indeed, when the classification of TO states is constructed using ordinary (clear) sets, the problem proves difficult to solve, since it is equivalent to establishing isomorphism between two non-isomorphic structures. On the one hand, there is a structure generated by a similarity relation, which in the general case is a non-transitive binary relation. In other words, for TO states w_1,w_2 and w_3, from " w_1 is similar to w_2" and " w_2 is similar to w_3" it does not always follow that " w_1 is similar to w_3". On the other hand, the structure is determined by the ambiguous logic of membership of ordinary (clear) sets, which defines the transitive relation between pairs of states as follows: from " γ_1 belongs to the same class as γ_2" and " γ_2 belongs to the same class as γ_3", it always follows that " γ_1 belongs to the same class as γ_3"

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For this reason, solving the problem of classifying situations that arise when choosing TO management within the framework of classical set theory is impossible or feasible only with a significant simplification of the problem. This is because the data describing a particular TO state are the results of "fuzzy mixing" of different sets. Traditional classification algorithms assign unique names to data groups – labels of the corresponding classes. For most real-life situations in TZA technology, such an approximation is not justified. A more realistic approach is to describe "fuzzily mixed" data using fuzzy sets.

From a systems perspective, the need to use fuzzy classification models when solving TO state classification problems follows from the Bellman-Zadeh incompatibility principle [7]. The essence of the principle is that high accuracy of analysis is incompatible with high complexity of the system under study. In other words, the more complex the system, the less we are able to give an accurate and practical judgment about its behavior. For systems whose complexity exceeds a certain threshold level, the accuracy and practical content of their models become concepts that are almost mutually exclusive. We can safely say that we have a certain analogue of the Heisenberg uncertainty principle in quantum mechanics.

This contradiction can be overcome within the framework of multi-valued (fuzzy) logic. A significant advantage of the fuzzy approach to classifying TO states is that, within the framework of multi-valued logic, meaningful solutions can be found for a wider class of problems than with a clear formulation. Indeed, if in conventional classification models an object $\boldsymbol{x_i} \in \boldsymbol{X}$ can either belong or not belong to a class $\boldsymbol{s_j}$, i.e., the degree of membership can only take two values – 0 or 1, then in fuzzy classification models the values of the degree of membership continuously cover the entire interval [0,1].

The principle of situational control of a complex system using the classification of its states was taken as the basis for the developed method of forming TO control models. Its essence is as follows.

Let there be *n* experimental data $\boldsymbol{x_1, x_2, \dots, x_n}$ for *m* input parameters, and *X* is the set of this data, i.e.

$$\boldsymbol{X = \{x_1, x_2, \dots, x_n\}; \quad x_k \in R^m; \quad k = 1, \dots, n.}$$

For fuzzy classification of available data, we use a typical scheme for extending clear classification models to a continuous case. Consider the division of *X* into *N* clusters. The value of *N* corresponds to the number of possible control modes $(\boldsymbol{u_1, u_2, \dots, u_l})$ and is selected heuristically at the initial stage, based on control constraints and the characteristics of the technological equipment.

Let us denote the degree of membership of x_k to the *j*-th fuzzy cluster by $\boldsymbol{z_{jk}}$, where the following must be satisfied:

$$\boldsymbol{z_{jk} \in [0, 1]; \quad \sum_j z_{jk} = 1.} \tag{4}$$

Fuzzy clustering allows data to belong to two or more clusters, but the sum of the degrees of membership is 1, and $\boldsymbol{z_{jk}}$ is the weight of membership in the cluster.

Let *Z* denote a matrix which elements are $\boldsymbol{z_{jk}}$, satisfying (4). Then clustering consists of a procedure for combining the data set *X* and the partition matrix *Z*. Let the result of the combination be denoted by Z_x. When determining the optimal Z_x, we will take the sum of quadratic errors in the generalized group as the objective function:

$$\boldsymbol{J(Z, c) = \sum_{k=1}^{n} \sum_{j=1}^{N} z_{jk} \left\| x_k - c_j \right\|^2}, \tag{5}$$

where $\boldsymbol{c_j}$ – center of the *j*-th cluster, $\boldsymbol{\dim c_j = \dim x_k = m}$;

||... || – a norm that reflects the similarity of the measured data and the cluster center. The norm can be taken as the Hamming distance or the Euclidean distance.

The values $\boldsymbol{z_{jk}}$ and $\boldsymbol{c_j}$, at which the value of the objective function (5) is minimal satisfy the conditions:

$$\boldsymbol{\hat{z}_{jk}} = \left(\sum_{l=1}^{N} \frac{\|x_k - \hat{c}_j\|}{\|x_k - \hat{c}_l\|} \right)^{-1}, \quad \boldsymbol{\forall j, k} \tag{6}$$

$$\hat{c}_j = \frac{\sum_{k=1}^{n} \hat{z}_{jk} \cdot x_k}{\sum_{k=1}^{n} \hat{z}_{jk}}, \quad \boldsymbol{\forall j.} \tag{7}$$

The value $\boldsymbol{\hat{z}_{jk}}$ that provides the minimum (5) is found using the following iterative procedure:

- set the initial value $Z^{(0)}$, chosen at random;
- calculate the cluster center $\boldsymbol{c_j^{(0)}}$ using $Z^{(0)}$ and formula (7);
- determine $Z^{(1)}$ using $\boldsymbol{c_j^{(0)}}$ and formula (6);

- set the threshold value ε and perform the previous steps until $\left\|Z^{(q)} - Z^{(q-1)}\right\| \leq \varepsilon$, $\left\|Z^{(q)} - Z^{(q-1)}\right\| = \max_{j,k}\left|z_{jk}^{(q)} - z_{jk}^{(q-1)}\right|$.

The adequacy of the obtained classification of TO states to real conditions is established by testing the classification model for stability. It is logical to consider the model that has maximum stability across the entire set of permissible modifications of the sample X as the most reasonable classification model.

Indeed, random classifications of experimental observations will usually be unstable, as they reflect the results of only one individual experiment. At the same time, if the classification model reflects the real patterns present in the analyzed data, it should be repeated in most experiments. As a result, the stability of such a classification (over the set of admissible modifications of the initial sample) will always be higher than the stability of a random, inadequate classification.

The standard approach to testing the stability of classification and forming admissible modifications of the set X is to randomly divide X into subsets and test the "performance" of the classification on these subsets [8]. However, the applicability of this approach to analyzing the stability of classification models is significantly limited by the requirement for the size of the experimental sample. It must be at least large enough to ensure that all "sub-samples" are representative. At the same time, in conditions of uncertainty, the actual volume of experimental information about TO is so small that its significant reduction can lead to a loss of representativeness of the samples obtained. Practice shows that the use of methods for managing the initial sample based on its reduction in fuzzy classification models is, as a rule, ineffective [9,10]. Therefore, let us consider an alternative approach to managing the experimental sample, namely, modifying the well-known R-algorithm [11].

Let us denote the initial sample of experimental data by X^1. Based on the experience and knowledge of the technologist, we will construct a sample X^2 such that card $X^2 \approx$ card X^1. Let $X^3 = X^1 \cup X^2$. To verify the stability of the obtained classification of sample X^1 into N clusters, the following approach is proposed.

We will use the procedure for constructing a matrix $Z_N^1 = Z$ for the set $X^1 = X$ and construct the corresponding matrix Z_N^2 for the set X^2 and Z_N^3 for the set X^3. We will form functions $\mu_j(k) = \left\{Z_{jk}^{(3)}\right\}$, where $Z_{jk}^{(3)}$ are the elements of the matrix Z_N^3. We will construct the narrowing of functions $\mu_j(k)$ on the sets X^1 and X^2. We will form the matrices and corresponding to these narrowings.

Let's calculate the values of the relative stability indicators of classification:

$$\beta_N^1 = \rho\left(Z_{1N}, Z_N^1\right);$$
$$\beta_N^2 = \rho\left(Z_{2N}, Z_N^2\right);$$

where $\rho\left(Z_{\alpha N}, Z_N^\alpha\right) = \frac{1}{2}\sum_{t,g=1}^{M^\alpha}\left|\delta_{tg} - \gamma_{tg}\right|$;

δ_{tg} and γ_{tg} – elements of adjacency matrices V_1 and V_2 of partitions $z_{\alpha N}$ and z_N^α $(\alpha = 1, 2)$, respectively;
$M^\alpha = Card\, X^\alpha$.

In this case, the following ratio applies:

$$V_1 = Z_{\alpha N} \cdot (Z_{\alpha N})^T; \quad V_2 = Z_N^\alpha \cdot (Z_N^\alpha)^T.$$

Here, the fuzzy product of matrices of blurred partitions is used as a matrix composition operation. This means that if A and B are matrices with elements a_{ij} and b_{ij}, then the elements of matrix C = A · B are calculated according to the rule

$$c_{ij} = \sum_{j=1}^{N} a_{ij} \wedge b_{ij}, \quad i, j = 1, \dots, n.$$

According to [5], only this interpretation of the matrix composition operation of fuzzy partitions guarantees the fulfillment of the equalities $\delta_{ii} = 1$, $\gamma_{ii} = 1$, $i = 1, \dots, M^\alpha$; $\alpha = 1, 2$.

The absolute classification stability index is calculated from the relative stability indices obtained:

$$\beta_N = \beta_N^1 + \beta_N^2.$$

Then, the ε-stability of the classification is evaluated (where ε is a predetermined number). If the condition $\beta_N \leq \varepsilon$ is met, the classification obtained is stable. If this condition is not met, further actions should be aimed at refining the a priori ideas about the value of N and the structure of the experimental sample.

If several different values of N are proposed a priori, then the classification with the minimum value β_N is considered the most stable (when the stability condition is met).

After checking the stability of the classification of maintenance states, a mapping f : X → P is formed. The mapping *f* is formed based on available a priori information about the technological process. The correctness of the control selection is checked using models (1) – (2) [12].

When selecting a technological mode for the current state $\boldsymbol{x_k}$, its belonging to one of the clusters $\boldsymbol{s_j}$ is determined by the maximum value $\boldsymbol{z_{jk}}$. Then the appropriate control is selected.

The main stages of the method involve the use of heuristic procedures.

3. CONCLUSIONS

The article presents a method for forming management models based on fuzzy classification of the states of technological objects. The proposed method for forming a TO management model is designed to solve multidimensional problems for use in conditions of a priori insufficiency and/or fuzziness of available information about the functioning of technological objects and the properties of external influences.

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