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MATHEMATICAL PHYSICS METHODS IN MODEL PROBLEMS OF RECEIVE AND TRANSMISSION BY SLOT BROADBAND ANTENNAS

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ABSTRACT

Electromagnetic rigorous formulation boundary problem for a semiinfinite slotted semitransparent cone is considered. Sources of electromagnetic field are radial electric or magnetic dipoles. Slots are periodically cut along the cone generatrices. The solution method combines the Kontorovich-Lebedev integral transformations and a singular integral equation (SIE) method. Analytical and numerical solutions are given. Semitransparent surface influence and a slot effect on basic electromagnetic characteristics are studied.

Keywords: cone, longitudinal, semitransparent, periodical, slot, integral equations.

1. INTRODUCTION

Conical antennas, due to their broadband and ultra-wideband properties, are integral elements of complex modern communication systems [1]. The process from design to operation of a real antenna is multi-stage, and one of the important stages in this chain is mathematical modeling. The works [2-7] are devoted to studying the features of model electromagnetic problems for conical structures. The solutions of model boundary value problems in a rigorous formulation for semiinfinite solid perfectly conducting cones and solid impedance ones are given in [3, 4, 6]. The presence of inhomogeneities on the cone surface, including slots, leads to the appearance of new physical effects, which significantly enriches the properties of the structures under consideration and expands the scope of their practical application. Some results of studying wave diffraction model problems for finite perfectly conducting solid cones and bicones, perfectly conducting cones with transverse and radials slots are given in the works [5, 7-8].

This paper presents the results of studying the model problem of exciting a semiinfinite semitransparent cone with periodic longitudinal slots by concentrated electromagnetic sources. The features of the surface parameter influence on main cone characteristics are found out.

2. PROBLEM STATEMENT. BASIC EQUATIONS

We consider a problem of exiting a semiinfinite circular infinitely thin semitransparent cone Σ with N periodical longitudinal slots by a harmonic radial electric ($\chi = 1$) or radial magnetic dipole ($\chi = 2$) that is located at the point B_0 (see Figure 1). The dipole has a moment $\vec{p}_r$ and its field changes in time as $e^{ia\omega t}$, $a = \pm 1$. Let us designate 2γ as an opening cone angle, $l = 2\pi/N$ as a structure period, d - as a slot width.

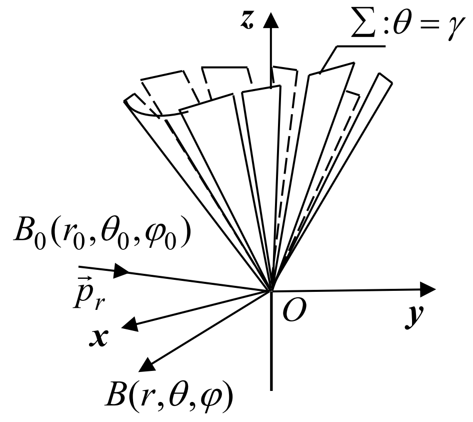

Figure 1. Geometric formulation of the problem

The structure period and the slot width are angular values those equal values of dihedral angle planes passing through the cone axis and the slot edges. In the spherical coordinate system r,θ,φ with the origin at the cone tip the slotted cone is defined as $\theta = \gamma$. The cone surface is capable of transmitting and reflecting a field with a semitransparency parameter W_χ $(W_\chi \geq 0)$. The task is to find the electromagnetic field $\vec{E}(\vec{r}),\vec{H}(\vec{r})$, that satisfies:

1) Maxwell's equations everywhere outside the cone semitransparent strips and the dipole;

2) boundary conditions at the conical strips:

$$\vec{n}\times\left\{\vec{n}\times L^{\chi-1}\left(\vec{E}^{+}+\vec{E}^{-}\right)\right\} = 2M^{(\chi)}\vec{n}\times L^{2-\chi}\left(\vec{H}^{+}-\vec{H}^{-}\right),$$

$L^0 = I$ is a unit operator,

$$L^{11}q = \left(\frac{\partial^2}{\partial r^2} - q^2\right)(rq),\ q = iak,\ \vec{E}^{\pm} = \vec{E}\Big|_{\theta=\gamma\pm 0};$$

3) the infinity condition.

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4) the limited energy condition.

Conditions 1)-4) guarantee the uniqueness of the problem solution [6]. Electromagnetic field components can be expressed via a scalar function (a potential) $\upsilon^{(\chi)}$ which satisfies:

a) the homogeneous Helmholtz equation everywhere outside the cone and the source;

b) the boundary condition at the conical semitransparent strips:

$$\frac{\partial^{\chi-1}}{\partial\theta^{\chi-1}}\upsilon^{(\chi)}\bigg|_{\Sigma} = 4^{1-\chi}\left(W_{\chi}\sin\gamma\right)^{\hat{\rho}(\chi)}\left[\frac{\partial^{2-\chi}}{\partial\theta^{2-\chi}}\upsilon^{(\chi)}\right]\bigg|_{\Sigma}, \quad (1)$$

$$[\tilde{g}]\big|_{\Sigma} = \tilde{g}\big|_{\theta=\gamma+0} - \tilde{g}\big|_{\theta=\gamma-0}, \quad \tilde{\rho}(\chi) = (-1)^{\chi-1};$$

c) the infinity condition;

d) the limited energy condition: $\int_V \left(\left|\upsilon^{(\chi)}\right|^2 + \left|\nabla\upsilon^{(\chi)}\right|^2\right)dv < +\infty.$

We seek the potential $\upsilon^{(\chi)}$ in the form

$$\upsilon^{(\chi)} = \upsilon_0^{(\chi)} + \upsilon_1^{(\chi)}, \quad (2)$$

where $\upsilon_0^{(\chi)}$ is a potential for the dipole field, and $\upsilon_1^{(\chi)}$ corresponds to the cone presence field.

The mathematical physics boundary problem a)-d) has the unique solution that can be found by applying the Kontorovich-Lebedev integral transformation with respect to the radial variable [6-7, 9]

$$\upsilon_1^{(\chi)} = \frac{2}{\pi^2}\int_0^{+\infty}\tau\,\mathrm{sh}\pi\tau\,\tilde{\upsilon}_{1,\tau}^{(\chi)}(\theta,\varphi)\frac{K_{i\tau}(qr)}{\sqrt{r}}d\tau, \quad (3)$$

$$\tilde{\upsilon}_{1,\tau}^{(\chi)} = \sum_{m=-\infty}^{+\infty} a_{m,\tau}^{(\chi)}\tilde{U}_{m,\tau}^{(\chi)}(\theta,\varphi),$$

$$\tilde{U}_{m,\tau}^{(\chi)} = \sum_{n=-\infty}^{+\infty} x_{m,n+m_0}^{(\chi)}\frac{P_{-1/2+i\tau}^{m+nN}(\pm\cos\theta)}{\dfrac{d^{\chi-1}}{d\gamma^{\chi-1}}P_{-1/2+i\tau}^{m+nN}(\pm\cos\gamma)}\times e^{i(m+nN)\varphi}, \quad (4)$$

here $q = iak$, k is a wave number, $K_{i\tau}(qr)$ is the Mackdonald function, $a_{m,\tau}^{(\chi)}$ are known coefficients, $P_{-1/2+i\tau}^{m+nN}(\pm\cos\theta)$ is the Legendre function (the sign "$+$" in the argument of the Legendre function corresponds to the region $0<\theta<\gamma$, and the sign "$-$" corresponds to the domain $\gamma<\theta<\pi$); $m/N = m_0+\nu$, m_0 is the nearest integer number to m/N, $-1/2 \le \nu < 1/2$. Let's use the boundary condition (1) and the continuity condition for $\upsilon_1^{(\chi)}$ (2), (3) to obtain the functional equations to find unknown coefficients $x_{m,n+m_0}^{(\chi)}$:

$$\sum_{n=-\infty}^{+\infty}\left\{1+W_{\chi}^{\tilde{\rho}(\chi)}\cdot 2^{3-2\chi}\left[N(n+\nu)\right]^{\tilde{\rho}(\chi)}\times\frac{|n|}{n}(1-\varepsilon_n^{(\chi)})\right\}x_n^{(\chi)}e^{inN\varphi} = e^{im_0N\varphi}, \ \pi d/l < |N\varphi| \le \pi, \quad (5)$$

$$\sum_{n=-\infty}^{+\infty}\left[N(n+\nu)\right]^{\tilde{\rho}(\chi)}\frac{|n|}{n}(1-\varepsilon_n^{(\chi)})x_n^{(\chi)}e^{inN\varphi} = 0, \ |N\varphi| < \pi d/l, \quad (6)$$

For an isotropic ($d=0$) semitransparent cone $U_{m,\tau}^{(\chi)}$ (4) is of the following form

$$U_{m,\tau}^{(\chi)} = \sum_{n=-\infty}^{+\infty}\frac{1}{1+W_{\chi}^{\tilde{\rho}(\chi)}\cdot 2^{3-2\chi}m^{\tilde{\rho}(\chi)}\dfrac{|m|}{m}(1-\varepsilon_{m,m}^{(\chi)})}\times\frac{P_{-1/2+i\tau}^{m+nN}(\pm\cos\theta)}{\dfrac{d^{\chi-1}}{d\gamma^{\chi-1}}P_{-1/2+i\tau}^{m+nN}(\pm\cos\gamma)}e^{i(m+nN)\varphi}.$$

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In the most interesting case for practical applications of axisymmetric ($\theta_0 = \pi,\ \chi = 1$) excitation and the source being located close to the top of a solid semitransparent cone ($kr_0 << 1$), the approximation for one of the electric field components is determined as follows

$$E_{1\theta}^{*} = \frac{ikp_1}{4W_1}\left(\frac{kr_0}{2}\right)^{-3/2+\tilde{\mu}_0} \frac{\sin kr_0}{r_0} \cdot \frac{e^{-ikr}}{r} \tan\frac{\theta}{2}, \qquad (7)$$

The physical meaning of (7) is that (7) is the electric field component of a spherical TEM wave, which propagates along the surface of a semitransparent cone from its top. Figure 2 shows the dependence of the value μ_0 on the parameters of the semitransparent cone. There is no this wave in the perfectly conducting solid cone structure. Near the semitransparent cone axis and outside the cone, the field of this wave is small and the field maximum is achieved on the very cone surface. The approximation for the radial component of the Umov–Poynting vector has the form ($kr_0 << 1,\ W_1 >> 1$)

$$S_r = \frac{\beta}{W_1^2 r^2} \tan^2\frac{\theta}{2}, \quad r > r_0,\ 0 < \theta < \gamma,$$

where β is a known coefficient.

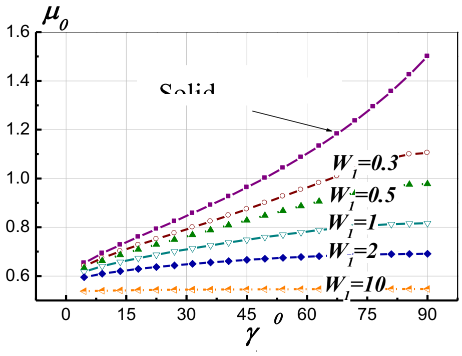

Figure 2. Dependence of on the half-opening angle of the cone for different values of the parameter W_1

For a perfectly conducting cone with longitudinal slots the solution of functional equations (5), (6) is equivalent to the solution of the following SIE that is defined at the slot ($\chi = 2$):

$$\frac{1}{\pi}\int_{CS} \ln\left|2\sin\frac{\psi-\beta}{2}\right| \Phi(\beta)\,d\beta + \frac{1}{2\pi}\int_{CS}\left[K(\psi-\beta) + D_{i\tau}\right]\Phi(\beta)\,d\beta = -D_{i\tau},\ \ \psi = N\varphi,\ \ |\psi| < \frac{\pi d}{l}, \qquad (8)$$

$$\Phi(\psi) = \sum_{n=-\infty}^{+\infty} y_n^{(2)} e^{in\psi},\ \psi \in [-\pi,\pi],\ K(\alpha) = \sum_{n\neq 0}\frac{1}{N|n|}\varepsilon_n^{(2)} e^{in\alpha},\ y_n^{(2)} = x_n^{(2)} - \delta_n^p,\ \delta_n^p = \begin{cases} 0, & n \neq p, \\ 1, & n = p, \end{cases}$$

$$D_{i\tau} = -\frac{\operatorname{ch}\pi\tau}{\pi\sin^2\gamma}\frac{1}{\frac{d}{d\gamma}P_{-1/2+i\tau}(\cos\gamma)\frac{d}{d\gamma}P_{-1/2+i\tau}(-\cos\gamma)}.$$

The singular integral equation (8) can be numerically solved by the method of discreet singularities [10-11]. In the case of the source being close to the cone top ($qr_0 << 1$), a single-mode approximation based on the solution can be used to analyze the scattered field. The approximation for the magnetic field component with axisymmetric excitation ($\chi = 2$, $\theta_0 = \pi$) of a perfectly conducting cone with narrow slots ($d/l << 1$) is of the form

$$H_{\theta,1}^{*} = \frac{1}{\sin^2\gamma\,\frac{1}{N}\ln\frac{1-u}{2}}\,\bar{\bar{A}}_1\left(\frac{qr_0}{2}\right)^{-1+\beta}\frac{\operatorname{sh}qr_0}{r_0}F^{*}(\theta,\varphi)\frac{e^{-qr}}{r}, \qquad (9)$$

$$F^{*}(\theta,\varphi) = c_1 + \frac{\sin\gamma}{\sin\theta}\left\{-1 + \operatorname{Re}\left[\frac{1 + c_N e^{iN\varphi}}{\sqrt{c_N^2 e^{2iN\varphi} - 2c_N e^{iN\varphi}\cos\delta + 1}}\right]\right\},\ \ \gamma < \theta < \pi,$$

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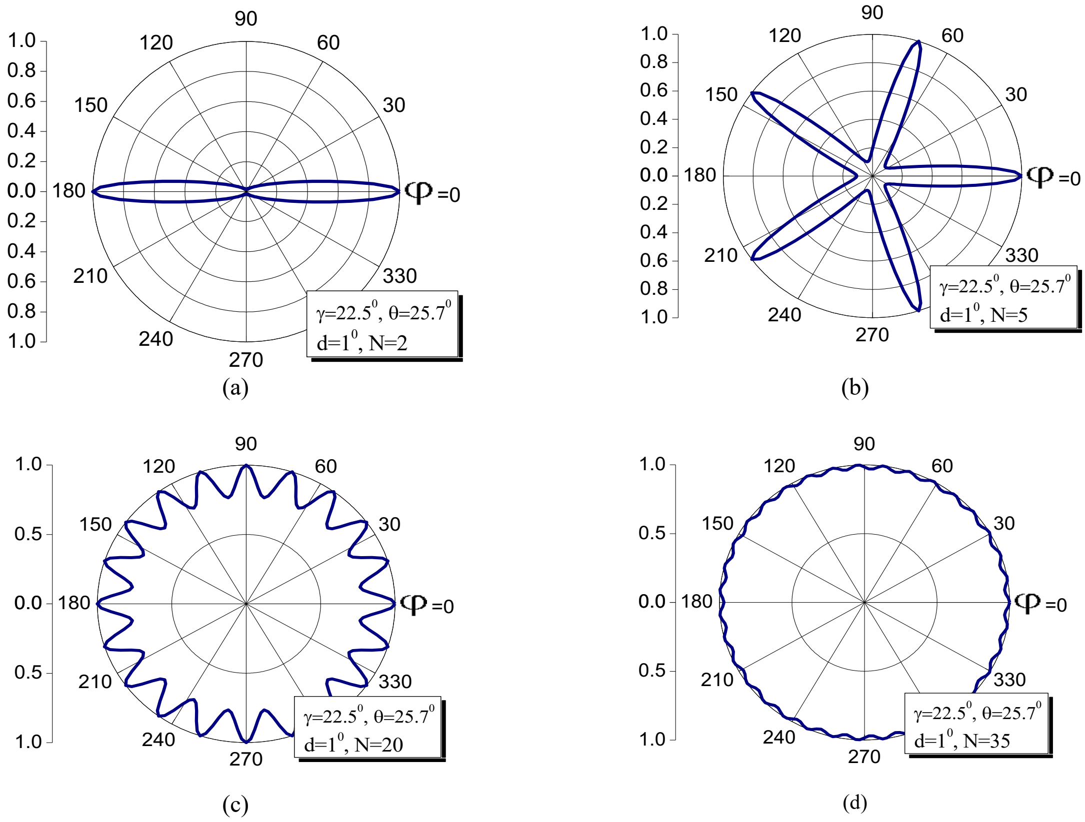

Figure 3. Normalized spatial distribution of the field in the single-mode approximation for a cone with narrow slots

here $u=\cos\delta$, $c_N=\left(\mathrm{ctg}\frac{\theta}{2}\Big/\mathrm{ctg}\frac{\gamma}{2}\right)^N$, $\delta=\frac{\pi d}{l}$, $\bar{\bar{A}}_1$ is a known coefficient,

$$\beta=\frac{1}{\left(-\frac{1}{N}\ln\frac{1-u}{2}\right)\sin^2\gamma}+O(N^{-2}\ln^{-2}(1-u)).$$

The approximation (9) corresponds to the field of TEM waves that propagate along the surface $\theta=\gamma$. The distribution of the TEM wave field in the azimuthal plane at different angles and the half-opening angle are given in Figure 3. In the single-mode approximation ($qr_0<<1$), the field of TEM waves is concentrated near the slots. Patterns of the spatial distribution of the TEM wave field in the azimuthal plane depending on the number of narrow slots are shown in Figure 3 (a), (b). These patterns have N - lobe character and with increasing a number of slots, approach the shape for a solid perfectly conducting cone in the case of its axisymmetric excitation, Figure 3 (c), (d). To study the spatial distribution of the field in cases of a non-narrow slot ($N=1$, $\chi=2$, $\theta_0=\pi$), the following asymptotic approximations ($qr>>1$, $\theta>2\gamma$) for the magnetic field components are used

$$\breve{H}_\theta=\frac{e^{-qr}}{r}\breve{F}_\theta(\theta,\varphi,\gamma,d),$$

$$\breve{F}_\theta(\theta,\varphi,\gamma,d)=\breve{\bar{A}}\int_0^{+\infty}\tau\,\mathrm{th}\pi\tau K_{i\tau}(qr_0)\frac{d}{d\gamma}P_{-1/2+i\tau}(\cos\gamma)\times\left(2\sum_{n=1}^{+\infty}x_n^{(2)}(\tau)\frac{\frac{d}{d\theta}P^{-n}_{-1/2+i\tau}(-\cos\theta)}{\frac{d}{d\gamma_2}P^{-n}_{-1/2+i\tau}(-\cos\gamma_2)}\cos n\varphi+\right.$$

$$\left.+x_0^{(2)}(\tau)\frac{P^{-1}_{-1/2+i\tau}(-\cos\theta)}{P^{-1}_{-1/2+i\tau}(-\cos\gamma)}\right)d\tau,\ \breve{H}_\varphi=\frac{e^{-qr}}{r}\breve{F}_\varphi(\theta,\varphi,\gamma,d),$$

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$$\breve{F}_{\varphi}(\theta,\varphi,\gamma,d)=\frac{\breve{\breve{B}}}{\sin\theta}\int_{0}^{+\infty}\tau\,\text{th}\,\pi\tau K_{i\tau}(qr_0)\frac{d}{d\gamma}P_{-1/2+i\tau}(\cos\gamma)\times\sum_{n=1}^{+\infty}x_n(\tau)\frac{P_{-1/2+i\tau}^{-n}(-\cos\theta)}{\frac{d}{d\gamma}P_{-1/2+i\tau}^{-n}(-\cos\gamma)}n\sin n\varphi d\tau,$$

where $\bar{\bar{A}}$ and $\breve{B}$ are known coefficients.

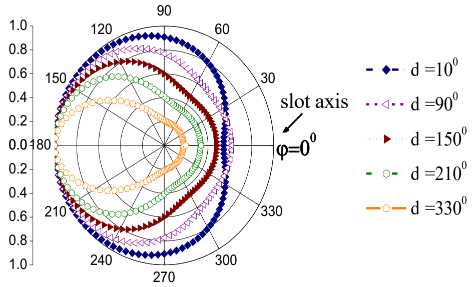

Figure 4. Field distribution patterns in the case of exciting a slotted cone ($\chi=2\ \ N=1,\theta_0=\pi,kr_0=1,\gamma=22.5^0,\theta=60^0$)

This made it possible to study the dependence of field distribution patterns on the parameters of a perfectly conducting cone with a longitudinal slot. Figure 4 shows patterns of the spatial distribution of the field at different slot width values. When the slot width is close to 10°, a small depression is observed in the middle of the slot. As the slot widens, the pattern shape takes on the shape of a conical strip pattern.

3. CONCLUSIONS

The paper presents the results of studying a rigorous formulation model problem of exciting a broadband loaded conical antenna with periodic longitudinal slots by concentrated sources. As a result of using the methods of Kontorovich-Lebedev integral transformations and singular integral equations, analytical and numerical problem solutions are obtained. The influence of the cone surface properties, as well as the presence and number of slots on the field patterns are studied. It is shown that the field patterns have a multilobe character in the case of a magnetic radial dipole location close to the top of a cone with narrow longitudinal slots. In this case, the reemitted electromagnetic field is concentrated near narrow slots of the conical structure. The field distribution patterns for a cone with one slot depending on the longitudinal slot width are given. The results obtained can be used in the design and creation of conical slot antennas, angular plane antennas, as well as radar reflectors.

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