# APPLICATION OF THE FUZZY SETS THEORY IN THE MANAGEMENT OF TECHNOLOGICAL PROCESSES FOR THE PRODUCTION OF AUTOMATION TECHNICAL MEANS

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#### **ABSTRACT**

It is established that the main problems of algorithmization for the management of technological processes under conditions of uncertainty are the high dimensionality of the task, the need for prompt identification of technological objects of control and the choice of optimal control influences. It is shown that these problems can be solved using fuzzy logic and decision theory.

A method for formalizing fuzzy concepts based on an objective probabilistic approach is proposed, which increases the reliability of modeling results. Based on fuzzy logic, a method for identifying multidimensional technological objects has been developed, in which, in order to reduce the dimensionality of the problem, a conjunctive inference rule is used instead of the generally accepted disjunctive one to build a relationship between a multidimensional input and output, which simplifies the inference procedure and leads to a more compact and convenient model for practical application.

Keywords: modeling, technological object, fuzzy environment, operational identification, problem dimension.

#### 1. INTRODUCTION

The urgency of the task of increasing the efficiency of technological process control (TP) under conditions of uncertainty has necessitated the development of an appropriate regularized apparatus. The analysis of the functioning of various control systems in industry based on fuzzy models has revealed the high efficiency of the new intelligent technology in creating flexible mobile control systems for complex poorly formalized industrial objects for various purposes. At the same time, studies have shown that the design methodology and software and information support tools for such systems to solve the problems of controlling the production of technical automation equipment (TAE) need to be improved.

#### 2. MODELING OF TECHNOLOGICAL OBJECTS IN A FUZZY ENVIRONMENT

One of the most important problems that ultimately determines the efficiency of managing modern technological objects (TOs) is the construction of an effective mathematical model. There are two main approaches to modeling in a fuzzy environment. The first approach is related to the construction of fuzzy functional dependencies or fuzzy functional systems, and the second is related to the construction of fuzzy relationships.

In practical situations, the dependence y = F(x) between the input  $x = (x_1, x_2, ... x_n)$  and the output of a maintenance facility is specified as a set of discrete data, as a certain family of points  $\{(x_i, y_i)\}$ , i = 1, ..., N. In the case of noisy or inaccurate data, the specification of F can only be approximate. Assuming some structural form of F, the available data can be used to identify the parameters of F. The usual approach to solving the identification problem in this case is to move from a fuzzy formulation of the problem to a clear one by using  $\alpha$ -level sets of fuzzy variables [1].

Thus, [2] considers the case when the dependence between these variables is represented in the form of a fuzzy linear regression equation with unknown parameters. In this case, it is assumed that  $x_i$  has deterministic values, and y has fuzzy values.

The task of identifying the TO is to estimate the parameters of the function

$$\tilde{y} = F(x_1, ... x_n, \tilde{a}_0, ... \tilde{a}_n) = \tilde{a}_0 + \tilde{a}_1 x_1 + \tilde{a}_2 x_2 + \dots + \tilde{a}_n x_n,$$
 (1)

where  $\sim$  is the fuzziness operator.

To estimate the parameters, we use the criterion of minimizing the deviation of the fuzzy values of the output parameter obtained by (1) from its sample fuzzy values  $\tilde{y}_i$ 

$$\widetilde{y} = \bigcup_{i=1}^{N} (\widetilde{y}_i - F(x_i, \widetilde{a}_0, \dots \widetilde{a}_n))^2 \to \min.$$
 (2)

By defining  $\alpha$ -level ( $\alpha \in [0,1]$ ) sets of fuzzy coefficients  $\widetilde{\alpha}_i$ , for each level  $\alpha_j$  (j=1,...,m), we obtain a regression equation similar to (1), but with clear coefficients. Thus, the original problem of estimating the



coefficients of the fuzzy regression equation (1) is reduced to the classical problem of estimating the parameters of multiple regression.

When structural identification of TOs is required, representations of the desired mappings are usually built using fuzzy granules [3].

Consider the data consisting of  $\{(x_i, y_i)\}$ , i = 1, ..., N, where  $X_i = \{(x, \mu_{X_i}(x))\}$  and  $Y_i = \{(y, \mu_{Y_i}(y))\}$  are convex and normalized fuzzy sets on X and  $Y_i$ , respectively;  $\mu_{X_i}(x)$ ,  $\mu_{Y_i}(y)$  are the corresponding membership functions (MFs). For simplicity, we assume that  $X, Y \subset R^1$ . For any i, the set  $Y_i$  is the image of the set  $X_i$  under the mapping  $Y_i$ , i.e., the principle of continuation can be used

$$\forall i, \forall y \ \mu_{Y_i}(y) = \sup_{\mathbf{x} \in X} \mu_{X_i}(\mathbf{x}) \tag{3}$$

under the constraint y = F(x). Relationship (3) can be expressed in words as follows: the more x belongs to  $X_i$ , the more y belongs to  $Y_i$ . In other words, if F is narrowed to the carrier  $S(X_i)$  of the set  $X_i$ , then F is a mapping with a fuzzy definition domain  $X_i$  and a fuzzy value domain  $Y_i$ . The identification problem is to find a function F that satisfies the relation (3). Each pair of sets  $(X_i, Y_i)$  gives rise to a part  $F_i$  of the function F defined on  $S(X_i)$ . Then all parts of  $F_i$  must be combined into a single function F.

The set of conditions for the existence of distinct mappings F that underlie the granular specification  $\{(X_i, Y_i)\}$  is as follows

$$\begin{cases} \forall i, \forall x \in S(X_i), \Phi_i(x) = \left\{ y \middle| \mu_{Y_i}(y) = \mu_{X_i}(x) \right\} \neq 0 \,, \\ \forall i, \forall j \neq i, \forall x \in S\left(X_i \bigcap X_j\right) \neq 0, \Phi_i(x) \cap \Phi_j(y) \neq 0 \,. \end{cases}$$

Under these conditions, the function F can (not exclusively) be chosen from the following conditions:  $\forall i, \forall x \in S(X_i), F(x) \in \Phi_i(x)$ . If F does not exist, then a fuzzy relation R is constructed to represent the granular specification instead of a clear representation of F. From the point of view of general systems theory [4], the TO is represented as a Cartesian product  $D \subset X \times Y$ .

Generalization of (3) to a fuzzy representation leads to the problem of finding a fuzzy relation R on  $X \times Y$ , constructed from fuzzy relations R, such that

$$\forall y \ \mu_{Y_i}(y) = \max_{\mathbf{x} \in \mathbf{X}} \min\left(\mu_{X_i}(x), \mu_{R_i}(x, y)\right). \tag{4}$$

As a solution to (4), we can take the direct product  $R_i = X_i \times Y_i$ , (fuzzy granule). Then (4) will be written in a compact form as  $Y_i = X_i \times R_i$ , and the general solution is defined as

$$R = \bigcup_{i=1}^{N} R_i = \bigcup_{i=1}^{N} X_i \times Y_i ,$$

where U means union.

This approach is widely used in the fuzzy control literature [5].

Using some kind of multi-valued implication  $R_i$ , can also be defined as follows:

$$\mu_{R_i}(x, y) = \max \left( 1 - \mu_{X_i}(x), \mu_{Y_i}(y) \right),$$
  

$$\mu_{R_i}(x, y) = \min \left( 1, 1 - \mu_{X_i}(x) + \mu_{Y_i}(y) \right).$$

In this case, the overall solution is defined as

$$R = \bigcap_{i=1}^{N} R_i \,,$$

where  $\cap$  denotes the intersection.

Although the fuzzy representation of a fuzzy granular specification is the most appropriate in the face of structural uncertainty in the TO model, the analytical approach remains very attractive. This is because a fuzzy relation R on  $X \times Y$  can be equivalent to a fuzzy representation of F. This representation is convenient for generalized analytical operations such as integration and differentiation.



To obtain an analytical representation of the original point data set, we combine the fuzzy granules, obtain the fuzzy relation R, and then approximate it using (L-R)-type functions.

The effectiveness of using fuzzy models is largely determined by their adequacy to managed maintenance. This issue has been theoretically studied very little.

In [6], the issues of constructing a fuzzy relation that has the property of good mapping are considered, while imposing difficult-to-implement restrictions on the membership functions of a fuzzy relation.

In [2], conditions were obtained under which the composite inference rule (4) is strictly fulfilled, which ensures the minimum value of the adequacy criterion

$$J = \sum_{i=1}^{N} \int_{V} \left( \mu_{Y_{i}}(y) - \mu_{\hat{Y}_{i}}(y) \right)^{2} dy,$$

where  $\hat{Y}_i$  is the estimate obtained from (4).

Strict implementation of the compositional rule means that for every given  $X_i$ , it is true that  $Y_i = X_i \circ R$  where  $\circ$  is the sign of the maximal product operation. Necessary conditions for the adequacy of a fuzzy relation are: regularity of matrices R, normality of sets, and fulfillment of the conditions  $\bigcap_{i=1}^{N} X_i = \emptyset$ ,  $\bigcap_{i=1}^{N} Y_i = \emptyset$ . The results obtained are valid for a multidimensional TP if the rules have a nested structure. However, this condition is often not met.

Thus, obtaining identification conditions for fuzzy systems is relevant, since the known works do not fully reflect the requirements for the correctness of the results.

The fulfillment of the conditions for the identification of a fuzzy model, i.e., the conditions of uniqueness of the solution and stability with respect to the initial data, is ensured by the correct mathematical formulation of the identification problem. In general, the problem is decomposed into structural and parametric identification, i.e., selecting a class of structures containing the desired solution, setting a priori the range of acceptable values of the structure's parameters, and finding their specific values. For fuzzy technological systems in relations, structural identification is carried out by selecting the following parameters [7]: a set of fuzzy variables and the power of fuzzy input sets  $X_i = \{(x, \mu_{X_i}(x))\}$  and output sets  $Y_i = \{(y, \mu_{y_i}(y))\}$ , i = 1,..., N; the type of corresponding membership functions  $\mu_{X_i}(x), \mu_{y_i}(y)$ ; implication options  $X_i \to Y_i$ , i.e. the logic of constructing a fuzzy relationship matrix. Parametric identification is carried out by selecting the region of valid fuzzy relations  $\{R_D\}$  that ensure the correctness of the solution, and determining the results of measurements of the input and output parameters of a particular fuzzy relation and its corresponding matrix  $R \subset \{R_D\}$ .

In addition to the general problems of modeling objects in a fuzzy environment, the TP for the production of motor vehicles has its own problems:

- the problem of developing methods of operational identification to build an adequate mathematical model that quantifies the TP in terms of "input-output";
- the problem of dimensionality when building a "current" model, since the TP as a control object is characterized by a large number of input and output parameters.

In addition, the problem of model adequacy is closely related to the problem of reliability of modeling results due to the ambiguity of the membership functions. The following material is devoted to solving these problems.

### 3. METHODS OF FORMALIZING FUZZY CONCEPTS AND VARIABLES IN MODELS OF TECHNOLOGICAL OBJECTS

In TO models, fuzziness can be formalized in different ways.

There are the following main classification features of methods of formalizing fuzziness [1]:

- by the type of representation of a fuzzy subjective assessment of any value (fuzzy set));
- by the type of the area of the MF values;
- by the type of MF definition area;
- by the type of correspondence between the definition domain and the value domain (unambiguous, multi-valued);
  - based on the homogeneity or heterogeneity of the MF value range.

The variety of types of fuzzy sets that arises in this case opens up wide possibilities for their application in the models of TP management of the TMA production. Thus, one of the grounds for the classification of fuzzy sets is the type of the MF definition domain. It is usually assumed that the MF definition domain of a fuzzy set coincides with the base set X. However, often the accuracy of modeling a real process does not deteriorate if you define fuzzy sets on subsets of the universal set X. Moreover, this makes it possible to reflect the dynamics of changes in the base set in a particular decision-making situation, for example, reducing the full number of comparable alternatives in a time-sensitive choice problem, and thus reduce the overall dimensionality of the problem.



A review of various ways of formalizing fuzziness has shown that two main approaches are being developed in this direction. The first is based on generalizing the concept of an element belonging to a set, which leads to blurring of the boundaries of the set, and in the extreme case, to the appearance of an object with indefinite boundaries - a semi-set. The second approach involves describing fuzziness with the help of a hierarchy - a family of ordered clear sets.

In both cases, the existing non-stochastic uncertainties are formalized through the MF. Expert judgment is usually used to build the MF.

There are a number of methods for constructing a fuzzy set MF based on expert opinions. Two groups of methods can be distinguished: direct and indirect methods. Direct methods are determined by the fact that the expert directly determines the rules for determining the values of the MF. An example of direct methods is the direct setting of the MF by a table, formula, or example. In indirect methods, the values of the MF are selected in such a way as to satisfy pre-formulated conditions. Expert information is only input for further processing. Additional conditions can be imposed on both the type of information received and the processing procedure.

Obviously, when building a MF based on expert opinions, we risk making mistakes that can significantly affect the solution, since the initial uncertainty of the problem essentially turns into uncertainty, ambiguity of the MF task. It is the subjective nature of MF (regardless of their interpretation) that leads to a certain skepticism and calls into question the effectiveness of applying fuzzy methods in practice.

An urgent task is to develop various methodological approaches to the formalization of fuzzy concepts that will increase the reliability of modeling results.

One way to solve this problem is the so-called concept of decision stability. Its essence lies in the fact that instead of constructing a MF, it is enough to roughly estimate the ranges of its non-trivial change, and then evaluate the behavior of fuzzy decisions when the MF is arbitrarily varied in the obtained valid regions. At the same time, very general and weak constraints are imposed on the MF. The optimal solutions are selected based on the obtained stable solutions, i.e., solutions that are not sensitive to changes in the MF.

The practical implementation of the described approach encounters a number of purely technical difficulties due to the need to repeatedly solve the decision-making problem. Thus, in [3], the degree of affiliation is determined through subjective probabilities.

This paper proposes a method for formalizing fuzziness based on an objective probabilistic approach. To construct the MF, we use the beta distribution [8]. The density of the beta distribution, defined in the interval [a, b], is

$$f(x,\gamma,\eta,a,b) = \begin{cases} \frac{1}{b-a} \frac{G(\gamma+\eta)}{G(\gamma)G(\eta)} \left(\frac{x-a}{b-a}\right)^{\gamma-1} \left(1 - \frac{x-a}{b-a}\right)^{\eta-1}, \\ a \leq x \leq b, & 0 \leq \gamma, & 0 \leq \eta, \\ 0 \text{ in other cases} \end{cases}$$

Here  $\gamma$ ,  $\eta$  are the shape parameters, G(·) is the gamma function.

The choice of the beta distribution is explained by the fact that it can have different shapes and, as a result, is used to describe a large number of real random variables whose values are limited to a certain interval. An example of such a random variable is the proportion of defective products on a production line per day.

The beta distribution also describes estimates of the duration of technological operations. An optimistic (o), pessimistic (p), and most likely (m) estimate of the time required to complete the operation is selected. Based on this information, it is assumed that the operation completion time has a beta distribution in the interval [o, p]; the most probable value is m, and the mean square deviation (MSD) is equal to  $\frac{1}{6}(p-o)$ .

The beta distribution is also used in the following situation. Suppose that there are m independent observations of some parameter in the TO. Let's arrange these values in ascending order. Let $y_r$  and  $y_{n-s+1}$  be the r-e smallest and s-e largest values, respectively. Then the proportion of x values of the original population that lie between  $y_r$  and  $y_{n-s+1}$  has a beta distribution with parameters y = n - r - s - 1 and y = r + s, i.e.

$$f(x; n-r-s+1, r+s) = \frac{\Gamma(n+1)}{\Gamma(r+s)\Gamma(n-r-s+1)} x^{n-r-s} (1-x)^{r+s-1},$$
  
  $0 \le x \le 1.$ 

This result is true regardless of the shape of the distribution of the random variable y.

Obviously, we are in similar conditions when forming a fuzzy variable's MF. Usually, the interval of its change [a, b] and the most probable value m are known. Let's consider a random variable x that has a beta distribution on [a, b], and the mode of x is equal to m. Considering  $x' = \frac{x-a}{b-a}$ , , let's move to a random variable with a beta distribution on [0,1]. Accordingly, its mode is  $m' = \frac{m-a}{b-a}$ . Then the MSD of x' will be equal to  $s' = \frac{1}{6(b-a)}$ .



From [9] we have

$$\begin{cases} (s')^2 = \frac{\gamma \eta}{(\eta + \gamma)^2 (\gamma + \eta + 1)}, \\ \\ m' = \frac{\gamma - 1}{\eta + \gamma - 2}, \quad (\gamma, \eta > 1), \end{cases}$$

Where do we find the form parameters  $\gamma$ ,  $\eta$ .

If there is additional information about the fuzzy variable in the form of a vector of its possible clear values  $\{x_i\}$ , then the expressions can be used to evaluate  $\gamma$  and  $\eta$ 

$$\begin{cases} \hat{\eta} = \frac{1 - \overline{x}}{s^2} [\overline{x}(1 - \overline{x}) - s^2], \\ \hat{\gamma} = \frac{\overline{x}\hat{\eta}}{1 - \overline{x}}, \end{cases}$$

where  $\overline{x}$  and  $s^2$  are the sample mean and variance.

Having built the density of the distribution f(x), we normalize it

$$\mu(x) = \frac{f(x)}{\max_{\mathbf{x}} f(x)}.$$

The function  $\mu(x)$  is taken as the MF of the fuzzy variable. This approach allows us to obtain a parameterized family of MFs for a fuzzy variable. The choice of a particular MF is well aligned with the existing level of uncertainty (fuzziness).

The maximum level of uncertainty corresponds to the case when we cannot identify the most probable value among all the values of a fuzzy variable. In this case, we get a partial type of beta distribution - uniform (n = v = 1).

The proposed approach differs from the existing ones by less subjectivity in the choice of the MF. Thus, its use increases the reliability of decision-making under uncertainty. The method is easy to implement and can also be used in the presence of linguistic descriptions.

### 4. IDENTIFICATION OF MULTIDIMENSIONAL TECHNOLOGICAL OBJECTS UNDER CONDITIONS OF UNCERTAINTY

A technological object as an object of identification can be considered as an element that dynamically transforms a certain signal. However, in order to correctly reflect the behavior of a maintenance facility under conditions of uncertainty, it is useful to adopt a different point of view, according to which the change in the internal state of the maintenance facility under the influence of various factors is taken into account [10]. Such a view of identification is important in order to unify the theoretical approach and generalizations to nonlinear and nonautonomous TOs under uncertainty.

The description of a TP in the state space can be represented in general as follows:

$$X^{k+1} = A \cdot X^k \,, \tag{5}$$

where  $X^k$  and  $X^{k+1}$  are the states of the TO at the moments  $t^k$  and  $t^{k+1}$ ;

A - some operator.

The task of identifying the TO is to find an estimate of A\* such that the relation for  $(X^{k+1})^*$ , found from the model  $(X^{k+1})^* = A^* \cdot X^k$ , is satisfied:

$$M\{\rho[X^{k+1}, (X^{k+1})^*]\} \to \min_{A^*}$$
 (6)

As an operator A, let's take a fuzzy relation R that connects the input X and output Y of the maintenance facility. The adequacy of the model to the real process determines the quality of its management. Building an adequate model under conditions of uncertainty is one of the main problems of identifying the TP.

This problem becomes more complicated if the TO is multidimensional. However, a characteristic feature of the TMA production is the multidimensional maintenance. Let's consider a method of identifying multidimensional maintenance, which allows us to reduce the dimensionality of the problem.



Let the TO be described by m input parameters  $\{X_1, ..., X_m\}$ , each of which can take n values obtained experimentally, and one output parameter, and let the description of the TO be given in the form of productive

if 
$$X_{11}, ..., X_{1j}, ..., X_{1m}$$
, then  $Y_1$ , otherwise ... ... if  $X_{i1}, ..., X_{ij}, ..., X_{im}$ , then  $Y_i$ , otherwise ... ... if  $X_{n1}, ..., X_{nj}, ..., X_{nm}$ , then  $Y_n$ ,  $i = 1, ..., n$ ,  $j = 1, ..., m$ ,

where  $X_{ij} = \{(u_k, \mu_{X_{ij}}(u_k))\}$  is the *i-th* normalized fuzzy value of the *j-th* input parameter  $X_i; u_k \in U_i; k = 1, ..., K;$ 

 $Y_i = \{(v_l, \mu_{Y_i}(v_l))\}$  - is the *i-th* normalized fuzzy value of the *j-th* output parameter  $Y; v_l \in V; l = 1, ..., L;$   $U_j, V$  - change areas  $X_j$  and Y, respectively.

System (7) is formalized as a set of fuzzy relationship matrices  $\{R_{ij}\}$ , for which the MF is defined by the expression:

$$\mu_{R_{ij}}(u_k, v_l) = \Omega\left(\mu_{X_{ij}}(u_k), \mu_{Y_i}(v_l)\right),\tag{8}$$

where  $\Omega$  is a certain fuzzy implication operator that connects input and output parameters.

To calculate the values of the output parameter, a composite inference rule is used, the type of which depends on the type of operator (8). In fuzzy control, the relation  $R_{ij}$  usually has an MF defined as:

$$\mu_{R_{ij}}(u_k, v_l) = \min\left(\mu_{X_{ij}}(u_k), \mu_{Y_i}(v_l)\right) \tag{9}$$

and the inference rule is of the form:

$$Y_t = \bigcup_{i=1}^n \bigcap_{j=1}^m X_{tj} \circ R_{ij}, \tag{10}$$

where  $X_{tj}$  is the value of the parameter  $X_j$  in the *t-th* dimension (t = n + 1, ...).

When using implication, it is assumed in advance that a fuzzy specialized mapping  $\{X_i\} \to Y$  is treated as a "coarse mapping", the inverse of "coarsely injective", since when we have  $\{X_{i1}, ..., X_{ij}, ..., X_{im}\}$ , any other fuzzy value other than  $Y_i$  is excluded. The use of implication (9) does not impose any restrictions in advance and does not prevent the pairs  $(\{X_{ij}\}, Y_i)$  and  $(\{X_{ij}\}, Y_i' \neq Y_i)$  from being represented in (7), which reduces the quality of the model in the sense of criterion (6).

In addition, implication (9) corresponds to the direct product  $\{X_{ij}\} \times Y_i$ , which does not contain the idea of a causal relationship between  $\{X_{ij}\}$  and  $Y_i$ .

Using the directed interpretation of the existing granular specification, which allows for a causal relationship between  $\{X_{ij}\}$  and  $Y_i$ , we define  $R_{ij}$  as follows:

$$\mu_{R_{ij}}(u_k, v_l) = \max\left(1 - \mu_{X_{ij}}(u_k), \mu_{Y_i}(v_l)\right),\tag{11}$$

We should note the property  $R_{ij}$  defined in (11):

$$\bigcap_{j=1}^m X_{tj} \circ R_{ij} \supset Y_t,$$

This property is due to the partial overlap of  $X_{tj}$  and  $\overline{x}_{tj}$  (supplemented by  $X_{tj}$ ), which prevents the recovery of the original information for low membership degrees. However, this is not essential, since the region of maximum MF values is the most informative in defuzzification.



In [3], it is shown that

$$\mu_{Y_t'}(v_l) = max\left(0.5, \mu_{Y_t}(v_l)\right),\,$$

where  $Y'_t = \bigcap_{j=1}^m X_{tj} \circ R_{ij}$ .

The global relation defined on the family  $\{R_{ij}\}$  can be obtained by considering  $R_j = \bigcap_{i=1}^n R_{ij}$ . In this case, the general rule of derivation will look like this:

$$Y_t = \bigcap_{i=1}^n \bigcap_{j=1}^m X_{tj} \circ R_{ij} = \bigcap_{j=1}^m X_{tj} \circ R_j.$$
 (12)

Since the intersection of fuzzy sets  $R_j$ , is used in (12), m matrices of fuzzy relations are calculated in the output process instead of  $(n \times m)$  matrices when they are output based on rule (10), which dramatically reduces the amount of computation, simplifies the output procedure and leads to a more compact, and therefore more convenient model for practical use.

In this case, the requirement of model adequacy in the sense of criterion (6) is satisfied.

#### 5. CONCLUSIONS

The methodological aspects of applying the theory of fuzzy sets in the management of technical process of production of motor vehicle equipment, concerning the reliability, accuracy and stability of the obtained solutions, are considered. A method of formalizing fuzzy concepts based on an objective probabilistic approach is proposed, which allows to increase the reliability of modeling results. The basics of TP management in the production of vehicle equipment are proposed, including a method for identifying multidimensional technological objects using a conjunctive inference rule to build multidimensional matrices of fuzzy relations.

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